Azerbaijan Journal of High Performance Computing, Vol 5, Issue 1, 2022, pp. 143-164
https://doi.org/10.32010/26166127.2022.5.1.143.164

AzJHPC
Azerbaijan Journal of High Performance Computing

*Correspondence: Mohammad Manthouri, Shahed University, Tehran, Iran, mmanthouri@shahed.ac.ir

Fuzzy Sliding Mode Controller for SEIR Model of Epidemic Disease

Mohammad Azimnezhad[1], Mohammad Manthouri[2] and Mohammad Teshnehlab[3]

[1] *Science and Research Islamic Azad University, Tehran, Iran, azimnezhad@gmail.com*
[2] *Shahed University, Tehran, Iran, mmanthouri@shahed.ac.ir*
[3] *K.N. Toosi university, Tehran, Iran, teshnehlab@eetd.kntu.ac.ir*

Abstract

This paper proposes a vaccination approach based on robust control for the SEIR (susceptible plus exposed plus infectious plus recovered populations) model of epidemic diseases. First, a classic sliding mode controller is investigated based on the SEIR model. Next, fuzzy logic is utilized to better approximate the uncertainties in the SEIR system using the sliding mode controller. Therefore, the proposed controller is a fuzzy sliding mode controller, which, compared to the sliding mode controller, provides an appropriate estimation of systems' actual parameters and removes the chattering phenomenon from the control signal. The stability of the controlled system is guaranteed using the Lyapunov theory simulations in which the classical sliding mode and the proposed controllers are compared, Using data from previous articles. Simulation results show that the proposed controller eliminates the susceptible subpopulation, incubated disease, and infectious diseases, eradicating the disease. Comparison with other methods reveals the better efficiency of the proposed method.

Keyword: Epidemic Disease, SEIR Model, Sliding Mode Controller, Fuzzy Controller, Vaccination, Uncertainty, Lyapunov.

1. Introduction

In the past few years, epidemic disease prevalence (e.g., Ebola, H1N1, H7N9 influenza, and COVID-19) has caused physical, mental, and economic harm to societies and governments. This has led both private and public sections to pay more attention to this issue and perform more studies about how to control, observe and act in such situations (Yang, F., Liu, H., Qi, H., & Liu, X., 2016, December).

Failing to control the disease is perilous, especially when facing an increase in travelers worldwide. Mathematical models describing infectious disease populations play an essential role in understanding the outbreak and control of the disease in the long term. Numerous epidemic frequency models have been proposed and analyzed over recent years to predict the spread of infectious disease in a region (Ibeas, A., De La Sen, M., & Alonso-Quesada, S., 2014; Di Giamberardino, P., & Iacoviello, D., 2017).

Depending on the behavior of the epidemic and assuming that the incubation period is neglected, there can be two models: SIR and SEIR. One of these models divides the total population into subpopulations based on people's conditions. The first subpopulation is represented by S (meaning people susceptible to disease). The next is shown by E (exposed, meaning infected people, but symptoms are not apparent). The third is I (meaning infectious people). The last subpopulation is R (meaning people who are recovered or are immune to the disease). In addition, based on the fact that the immunity of recovered individuals to the disease is permanent or temporary, two other models can be considered: SIRS and SEIRS (McCluskey, C. C., 2010; Yi, N., Zhang, Q., Mao, K., Yang, D., & Li, Q., 2009).

The main objective of this paper is to design a robust vaccination strategy capable of eradicating infectious diseases in a population, regardless of unknown parameters' uncertainties of the disease.

The rest of this paper is structured as follows: Section 2, previous works are studied. Section 3 describes the SEIR epidemic model. Section 4 discusses the classic sliding mode controller. The fuzzy controller is described in section 5. Section 6 proposes the fuzzy sliding mode controller. Simulations and discussions to investigate the efficiency of the proposed controller are provided in section 7. Finally, section 8 concludes the paper.

2. Related Work

Studying epidemic diseases plays an essential role in the world as it provides scientists with more knowledge to manage diseases. Therefore, this section mentions recent studies using different methods based on control theory for epidemic disease models such as SIR and SEIR.

In (Ibeas, A., de la Sen, M., & Alonso-Quesada, S., 2014, December), Micken's discretizing method is used to obtain the discrete-time SEIR epidemic model. Then, an adaptive controller is devised to ensure that estimated parameters are non-negative and that the disease is thoroughly eradicated. A feedback-based vaccination strategy for the SEIR epidemic model is proposed in (Ibeas, A., de la Sen, M., Alonso-Quesada, S., & Nistal, R., 2015, May). This strategy hinges on the relative stability concept. In this vaccination strategy, the whole susceptible population does not need to converge to 0, and it can be under a certain threshold. Directly resulting from this method, not every person in the population needs to be vaccinated. Thus, this method is more economical. Yan Cheng has investigated the delayed version of the epidemic SEIR model in (Cheng, Y., Pan, Q., & He, M., 2013) with a nonlinear prevalence rate based on continuous treatment and impulsive vaccination. Seasonal weather changes can impact the infectious disease spread vastly. Therefore, a study of the epidemic disease SEIR model based on periodic impulsive vaccination and seasonal contact rate is presented in (Bai, Z., & Zhou, Y., 2012).

The vaccination strategy in (Alonso-Quesada, S., De la Sen, M., Agarwal, R. P., & Ibeas, A., 2012) is based on a time-continuous nonlinear control rule that linearizes the output-input feedback. An observer is fitted in the general control scheme, To provide an online estimate of the population prone to disease and diseased population when an online measurement is not feasible.

In (Jiao, H., & Shen, Q., 2020), an SEIR epidemic model was introduced to analyze the significant characteristics such as positivity and boundedness. Moreover, it proposed an approach to deal with the primary reproduction number. A sliding mode control strategy was also used to limit the number of infectious people to the desired value. However, model parameters are considered to be known. A fuzzy fractional strategy was used in (Dong, N. P., Long, H. V., & Khastan, A., 2020), in which epidemic models were investigated in the presence of uncertain parameters.

Asier Ibeas et al. have designed a sliding mode controller for the epidemic SEIR model (Ibeas, A., De La Sen, M., & Alonso-Quesada, S., 2014) In this paper, a controller is designed, supposing that specific knowledge about the upper limit of the uncertainty signal is acquired. Then, this condition is eliminated by designing an adaptive sliding mode control system. Furthermore, the SEIR epidemic model is developed based on a fractional degree dynamic (Ibeas, A., Shafi, M., Ishfaq, M., & Ali, M., 2017).

Choosing the design method of the controller depends on the controlled system and its definition of appropriate performance. Systems are usually nonlinear with uncertainties (Schoukens, J., & Ljung, L., 2019). These uncertainties can be caused by imprecise recognition of the controlled system and noise and distortion (Mattsson, P., Zachariah, D., & Stoica, P., 2018). Traditional nonlinear control methods are primarily based on a system model, whereas system identification is needed when the system model is non-existent. System identification might introduce uncertainties caused by the imprecise recognition of system parameters. Therefore, it is reasonable to use proper control methods, which are not dependent on the system model and are capable of adequately covering all uncertainties (G. Drakopoulos, P. Mylonas and S. Sioutas, 2019). This leads us to intelligent controllers such as fuzzy logic and neural networks. (Wu, S. J., & Lin, C. T., 2000). These controllers facilitate different abilities, including various structure designs based on needs, training, and expertise (Jang, J. S., 1992, March). Classic controllers' strong background in analysis and proof of stability on one hand and system model independency of the intelligent controllers makes it considerably tangible to combine these two types of controllers to obtain a superior controller (Jang, J. S., 1992, March; Lee, H., Kim, E., Kang, H. J., & Park, M., 1998). In (Manthouri, M., Aghajari, Z., & Safary, S., 2022), a reliable processing system for blood samples and classifying five types of white blood cells in microscopic images is designed. The Gram-Schmidt algorithm is used for segmentation purposes. Scale-Invariant Feature Transform (SIFT) feature detection technique with a deep convolutional neural network is applied to classify different types of white blood cells.

This paper proposes a combination of adaptive fuzzy controllers with a classic sliding mode, Considering un-modeled dynamics and the presence of uncertainties in unknown parameters. This way, the gain of the switching part of the system's sliding mode control rule, which corresponds to compensating the difference between actual and nominal values of the system, is estimated through fuzzy control. This is followed by a discussion over the improvement of the system's output, parameters' and functions' estimations, and a comparison to the classic sliding mode controller (Ohtake, H., Tanaka, K., & Wang, H. O., 2006). An analytic vaccination policy using only the inaccurate information of the infected is established for a susceptible-infected-recovered epidemic model based on Takagi-Sugeno (T-S) fuzzy model presented (Lee, H. J., 2022).

3. SEIR Model Characteristics

Various types of models have been used to study epidemic disease contagion. Figure (1) shows the block diagram of the SEIR epidemic disease dynamic model. Based on the SEIR model presented in [2], the population is divided into 4 groups. These four groups are as follows: population exposed to the disease (susceptible) S(t), population exposed to the disease whereas the disease is not infectious yet E(t), population with the infectious disease I(t), and recovered population R(t). The latter group corresponds to the population in which individuals have naturally managed to recuperate from the disease or through vaccination.

The SEIR model is described using the below equations:

$$\dot{S}(t) = -\mu S(t) + \omega R(t) - \varphi(S, E, I, R) + \nu N(t) - N(t)V(t) \tag{1}$$

$$\dot{E}(t) = \varphi(S, E, I, R) - (\mu + \sigma)E(t) \tag{2}$$

$$\dot{I}(t) = -(\mu + \gamma)I(t) + \sigma E(t) \tag{3}$$

$$\dot{R}(t) = -(\mu + \omega)R(t) + \gamma I(t) + N(t)V(t) \tag{4}$$

Where $X^T = [S\ E\ I\ R]$ is the state vector, N(t) shows the whole population at any given moment, μ is an indicator of mortality rate due to causes other than infectious diseases, v is a representative of birth rate, ω shows loss of health rate, α is saturation coefficient, and σ and γ correspond to mean incubation and infectious periods, respectively. φ is a nonlinear function representing the disease prevalence rate. When $\varphi(S, E, I, R) = \beta S(t)I(t)$, this function is called a two-way linear prevalence rate, and when $\varphi(S, E, I, R) = \beta S(t)I(t)/(1 + \alpha S(t))$, it is called to be a saturated prevalence rate. Here β represents the transmission rate. The standard prevalence rate is described as follows (in the literature, standard prevalence rate is the most commonly used rate):

$$\varphi(S, E, I, R) = \beta S(t)I(t)/N(t) \tag{5}$$

By summing equations 1 to 4, population changes are obtained as follows:

$$N(t) = S(t) + E(t) + I(t) + R(t) \tag{6}$$

$$\dot{N}(t) = (\nu - \mu)N(t) \tag{7}$$

The error signal can be defined as shown in equation 8, in which $N_{ref}(t)$

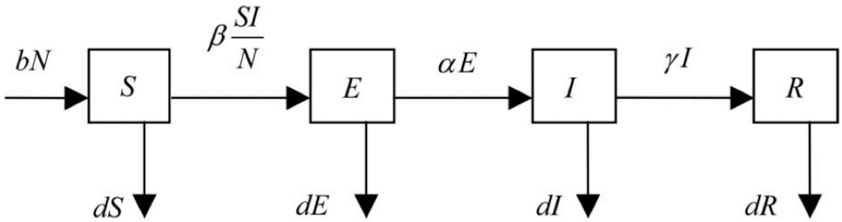

Fig. 1. Block diagram of the SEIR epidemic disease dynamic model (Ibeas, A., de la Sen, M., & Alonso-Quesada, S., 2014, December)

corresponds to the desired process of immunity of the population. In other words, $N_{ref}(t)$ is the reference signal.

$$e(t) = R(t) - N_{ref}(t) \tag{8}$$

$$N_{ref}(0) = R(0) \tag{9}$$

$$\left(R(t) - N_{ref}(t)\right) \to 0, t \to \infty \tag{10}$$

4. The Classic Sliding Mode Controller

In this section, we discuss the classic sliding mode controller. Sliding mode controlling is one of the most well-known methods due to design simplicity and generalizability, providing good results. The main idea behind the sliding mode controller is to simplify the nonlinear nth degree equations of the control problem to a control problem with nonlinear first-degree equations while considering parameters' uncertainties and system disturbance (Hung, J. Y., Gao, W., & Hung, J. C., 1993). Various structures on the two sides of the sliding surface in the nonlinear sliding mode controller make it capable of obtaining robust control features (Edwards, C., & Spurgeon, S., 1998). A single input non-autonomous dynamic open-loop system of order n could be given as (Tokat, S., Eksin, I., & Güzelkaya, M., 2003).

$$\dot{x}_j(t) = x_{j+1}(t) \quad (j = 1, \dots, n+1)$$
$$\dot{x}_n(t) = \sum_{i=1}^{q} (a_i + \Delta_i(t))\, f_i(x,t) + p(x,t)u(t) + d(t) \tag{11}$$

Where $X(t) = (x_1, x_2, \dots, x_n) \in R^{n\times 1}$ is the state vector, and $a_i\ (i = 1 \dots q)$ shows static system parameters. $\Delta_i(t)$ represents parameter perturbations with bounded uncertainties, $u(t)$ Shows the input signal, d(t) is a time-dependent disturbance with the known upper bound $f_i(x,t)(i = 1 \dots q)$ and $p(x,t)$ are functions that define system characteristics. Thus, the control problem is for X(t) to determine $X_d(t) = (x_{d1}(t), x_{d2}(t), \dots, x_{dn}(t))$ track. a general linear sliding surface for system (10) can be written as:

$$\phi(x,t) = \sum_{j=1}^{n} e_j(t) + \lambda_j e_j(t) \tag{12}$$

Where λ_j is a strictly positive real value. Track error can be defined as follows:

$$e(t) \triangleq \left(e_1(t)\,, \dots, e_n(t)\right) = \left(x_1(t) - x_{d1}(t)\,, x_n(t) - x_{dn}(t)\right) \tag{13}$$

Where X_{dn} is the nth state of the track. Equation 11 is a linear function of the system's error, and λ_j determines the sliding level. Assuming $\dot{e}_{(n-1)}(t) = e_n(t)$ we can consider $\dot{x}_{d(n-1)}(t) = x_{dn}(t)$ A homogenous differential equation with the unique answer of e=0 can be obtained by tuning $\phi = 0$. Therefore, the error asymptotically approaches 0 if a proper control rule is used. This rule must contain the track within the sliding level. The nominated function from the direct Lyapunov method can be defined as below:

$$V(s) = \frac{1}{2}\phi^2 \tag{14}$$

Where $V(s) > 0 \; and \; V(0) = 0$ for every $s \neq 0$. The aim here is to define a negative derivative for the Lyapunov function. A sufficient statement for system stability is given in the below inequality:

$$\dot{V} = \frac{1}{2}\frac{d}{dt}\phi^2 \leq \eta|\phi| \tag{15}$$

Where η is a determinable strictly positive number. The inequality in equation 15 implies that the system is stable and controlled. This means that the system's state always moves towards the sliding level. Thus, equation 15 is the condition required to reach the sliding level. Replacing equation 12 in equation 15, we obtain:

$$\phi.\dot{\phi} = \left(\sum_{j=1}^{n-1}\sum_{i=1}^{q}(a_i + \Delta_i)\, f_i + pu + d - \dot{x}_{dn} + \lambda_j\left(x_{j+1} - \dot{x}_{dj}\right)\right) \leq \eta|\phi| \tag{16}$$

In equation 16, a sigmoid function is achieved. Note that the sigmoid function definition is as in the following:

$$sign(x) = \begin{cases} -1 & if\ x < 0 \\ 0 & if\ x = 0 \\ 1 & if\ x > 0 \end{cases} \tag{17}$$

Uncertainties need to be limited to formulate an SMC rule such that:

$$\Delta^{-} \leq \Delta_i(t) \leq \Delta^{+} \tag{18}$$

$$\gamma^{-} \leq d(t) \leq \gamma^{+} \tag{19}$$

Control input that could satisfy the reach condition can be chosen as:

$$u = \underbrace{\left(-\sum_{j=1}^{n-1}\sum_{i=1}^{q} a_i f_i + \dot{x}_{dn} - \lambda_j\left(x_{j+1} - \dot{x}_{dj}\right)\right)/p}_{u_{eq}} - \underbrace{\left(k + \sum_{i=1}^{q} |\bar{\Delta}_i f_i|\right) sign(\phi)/p}_{u_{dis}} \quad (20)$$

A cautious choice for $\bar{\Delta}$ is:

$$\bar{\Delta} = max\{|\Delta^-|, |\Delta^+|\} \quad (21)$$

Considering the external disturbance, we can write the lower limit of K as:

$$k > \eta + max(|\gamma^-|, |\gamma^+|) \quad (22)$$

Where $k + \sum_{i=1}^{q} |\bar{\Delta}_i f_i|$ is the discontinuous control rule gain, this is a strictly positive actual function whose lower limit depends on systems' parameters' estimation and external disturbance. Control input in equation 20 comprises two parts. The first part is Ueq, a continuous term recognized as equivalent control based on systems' parameters. This term compensates for undesired dynamic estimation of the system. The second term is the sigmoid function of the discontinuous control rule. Udis needs unlimited switching at the cross-section of the error state track and sliding level for the control signal. Therefore, the track must constantly move toward the sliding level (Bartoszewicz, A.,1995).

5. Fuzzy Controller

In the past two decades, it has been shown that fuzzy systems and neural networks can be considered global function approximators and can correctly estimate almost any continuous function with a small and compact set. There are several advantages to using fuzzy systems compared to other nonlinear models. These advantages include understanding the structure, empirical and statistical knowledge, and interpretability. This section presents a fuzzy dynamic model using the TAKAGI-SUGENO (TS) as a global function approximator.

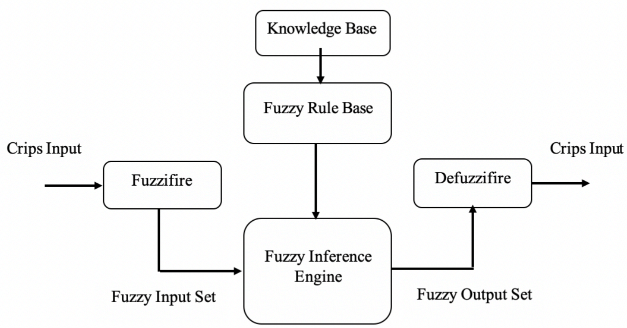

Fig. 2. Main structure of fuzzy systems

Figure 2 depicts the main structure of fuzzy systems. This structure comprises four segments; fuzzified, fuzzy rule base, fuzzy inference engine, and defuzzifire. The deterministic input is firstly fuzzified and normalized to fuzzy input sets. This stimulates and activates the fuzzy inference engine and fuzzy rule base, which outputs fuzzy sets. Finally, defuzzifiring these sets output deterministic output values.

This fuzzy method is based on "if-then" fuzzy rules to map input vector $X = [x_1, x_2, \dots, x_n]$ to output $\hat{g}(\underline{x}, \underline{\theta})$. A fuzzy rule base contains a set of "if-then" rules as below (Tokat, S., Eksin, I., & Güzelkaya, M., 2003): if x_1 F_1^m is and …, then $\hat{g}$ is θ_1^m،

Where $F_i^m i = 1, \dots, n$ are fuzzy variables based on fuzzy membership functions $\mu_{F_i^m}(x_i)$. θ_l^m relates to fuzzy output values. Fuzzy system output can be expressed based on individual fuzzified, multiplication inference, and mean center defuzzifire as below:

$$\hat{g}_l(\underline{x}, \underline{\theta}_l) = \frac{\sum_{j=1}^{m} \theta_i^j \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)}{\sum_{j=1}^{m} \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)} = \underline{\theta}_l^T \xi^l(x) \tag{23}$$

Where m is the total number of fuzzy rules and $\underline{\theta}_l = \left[\underline{\theta}_l^1, \underline{\theta}_l^2, \dots, \underline{\theta}_l^m \right]^T$ is the tuning parameters' vector.

$\xi^l(x) = [\xi_1^l(x), \xi_2^l(x), \dots, \xi_m^l(x)]^T$ is the fuzzy base function vector.

$$\xi_j^l(x) = \frac{\left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)}{\sum_{j=1}^{m} \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)} \tag{24}$$

Optimized parameters' vector and fuzzy estimation error are defined as below:

$$\underline{\theta}_l^* = \underset{\theta_l \in R}{\arg} \underset{\prod_{i=1}^{l} p_i}{\min} \left[\sup_{X \in R} \left| g_l(\underline{x}) - \hat{g}_l(\underline{x}, \underline{\theta}_i) \right| \right] \tag{25}$$

Also, $\hat{g}(x)$ tunable parameters can be tuned online using Lyapunov method.

6. Proposed Controller

This section uses a classic sliding mode controller for the SEIR model. Then, using fuzzy logic and online parameters tuning, the switching gain of the control rule of the sliding mode controller is estimated. This compensates for the uncertainty of the system. In addition, a comparison is made between the performance and robustness of the proposed fuzzy sliding mode controller and an SEIR model with parametric uncertainties.

6.1. Designing Fuzzy Sliding Mode Controller for SEIR Model

Here we represent how to design a fuzzy sliding mode controller for the SEIR model based on the one proposed in [2].

First, the sliding level is defined as follows. Note that λ is the constant positive gain.

$$\phi(t) = e(t) + \lambda \int_0^t e(\tau)d\tau \tag{26}$$

$$\phi(t) = e(t) + \lambda \int_0^t e(\tau)d\tau = 0 \tag{27}$$

The reference signal used in this paper for theoretical analysis and simulation is an exponential one which is defined as below:

$$N_{ref}(t) = (R(0) - N(0))e^{-\varepsilon t} + N(t) \tag{28}$$

It can be seen that satisfying equation 26 ensures achieving the control objective.

$$\begin{gathered}\dot{\phi}(t) = \dot{e}(t) + \lambda e(t)\\ = \dot{R}(t) - \dot{N}_{ref}(t) + \lambda R(t) - \lambda N_{ref}(t)\\ = -(\mu + \omega)R(t) + \gamma I(t) + N(t)V(t) - (\nu - \mu)N(t)\\ +\varepsilon e^{-\varepsilon t}\left(R(0) - N_{ref}(0)\right) + \lambda R(t) - \lambda N(t) - \lambda e^{-\varepsilon t}\left(R(0) - N_{ref}(0)\right)\end{gathered} \tag{29}$$

For simplicity, it is assumed $\varepsilon = \lambda$. This removes parts with the initial value.

$$= -(\mu + \omega)R(t) + \gamma I(t) + N(t)V(t) - (\nu - \mu)N(t) + \lambda R(t) - \lambda N(t) = 0 \tag{30}$$

Equivalent control rule can be obtained from the above equation:

$$V_{eq}(t) = \frac{1}{N(t)}[(\lambda + \nu - \mu)N(t) + (\mu + \omega - \lambda)R(t) - \gamma I(t)] \tag{31}$$

Since θ includes systems' actual parameters and is unknown, we use nominal and estimated values instead and show them as $\hat{\theta}$. To simplify the equations, we consider the below substitutions:

$$\begin{aligned}\delta = \lambda + \nu - \mu \quad &\rightarrow \quad \hat{\delta} = \lambda + \hat{\nu} - \hat{\mu}\\ \rho = \mu + \omega - \lambda \quad &\rightarrow \quad \hat{\rho} = \hat{\mu} + \hat{\omega} - \lambda\end{aligned} \tag{32}$$

$$V_{eq}(t) = \frac{1}{N(t)}\left[\hat{\delta}N(t) + \hat{\rho}R(t) - \hat{\gamma}I(t)\right] \tag{33}$$

We have considered an additional part in the control segment to compensate for the discrepancies between nominal and actual values in equation 31. This led to equation 33:

$$V = V_{eq}(t) - \frac{g(x,t)}{N(t)} sgn\big(\phi(t)\big) \tag{34}$$

Where g(t) represents the added switching's control gain, it is determined using uncertainties of the parameters.

Assumption 1: there exists a function depending on the system's state b(x,t), which bounds the upper limit of the below equation.

$$\left|(\hat{\rho} - \rho)R(t) + (\gamma - \hat{\gamma})I(t) + \left(\hat{\delta} - \delta\right)N(t)\right| \leq b(x,t) \tag{35}$$

Assumption 1 relates to maximum uncertainty caused by mis-coordination between parameters and their fundamental values. Note that b(x,t) always exists because a model is parametrized using determined values, although these parameters may not be known. In addition, this assumption incorporates an accurate epidemiologic interpretation. Equation 35 can be rewritten as follows:

$$\left|\hat{\delta}N(t) - \delta N(t) + \hat{\rho}\,R(t) - \rho R(t) + \hat{\gamma}I(t) - \gamma I(t)\right| \\ = \left|N(t)V_{eq}(t) - N(t)V_{eq.true}(t)\right| \rightarrow N(t)\left|V_{eq}(t) - V_{eq.true}(t)\right| \leq b(x,t) \tag{36}$$

Where $V_{eq}(t)$ Is the equivalent control rule obtained from equation (34) and is expressed in terms of nominal values of $\hat{\theta}$. $V_{eq.true}$; however, if the equivalent control rule with actual values. This assumption states that the absolute value of exerted vaccination control function when the parameters are known and not known is bounded by a deterministic function.

Assumption 2: upper limit of b(x,t) is known.

As mentioned earlier, this assumption means that the limitation is effectively known. This is a common assumption in sliding mode control systems. However, this assumption does not limit the possibility of using the control process since the upper limit of (35) can be calculated from the lower and upper limits of system parameters.

Assumption 3: switching gain g(x,t) is chosen as below.

$$g(x,t) = b(x,t) + \eta \tag{37}$$

Where $\eta > 0$ and is chosen arbitrarily by the designer.

Assumption 4: $\bar{b}$ is an unknown positive value such that:

$$\bar{b} \geq b(x,t) \tag{38}$$

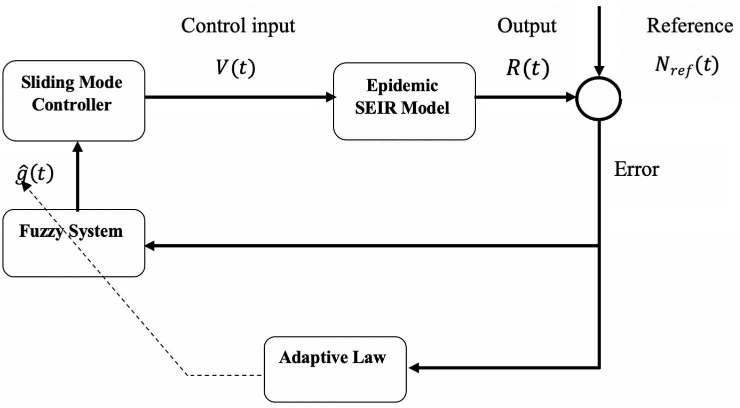

Fig. 3. Adaptive Fuzzy Sliding Mode Controller block diagram

Since the exact value of g(x,t) is unknown, its value is fuzzily estimated in the control rule equation.

$$V = V_{eq}(t) - \frac{\hat{g}(t)}{N(t)} sgn\big(\phi(t)\big) \tag{39}$$

$\tilde{g}(t) = \hat{g}(t) - g$ is the difference between actual and estimated values of g.

G is fuzzily estimated as follows:

$$\hat{g}(X|\theta) = \theta^T \xi(X) \tag{40}$$

And the optimized value of θ and ideal estimate of g is defined as follows:

$$\hat{g}(X|\theta^*) = {\theta^*}^T \xi(X) \tag{41}$$

Theorem:

The Lyapunov function is considered, To achieve general stability and adaption rule as below.

$$L(t) = \frac{1}{2}\phi(t)^2 + \frac{1}{2z}(\theta - \theta^*)^T(\theta - \theta^*) \tag{42}$$

The time derivative of equation 40 can be calculated as:

$$\dot{L}(t) = \phi(t)\dot{\phi}(t) + \frac{1}{z}(\theta - \theta^*)^T\dot{\theta} \tag{43}$$

Noting equation 29, we can rewrite equation 43:

$$= \phi(t)\left[(\hat{\rho} - \rho)R(t) + (\gamma - \hat{\gamma})I(t) + \left(\hat{\delta} - \delta\right)N(t) - \hat{g}(x,t)sgn(\phi)\right] + \frac{1}{z}(\theta - \theta^*)^T\dot{\theta} \tag{44}$$

$$\tilde{g}(t) = (\theta - \theta^*)^T \xi(X) + \varepsilon_f \tag{45}$$

$$\varepsilon_f = g - g(\theta^*|\xi(X)) \tag{46}$$

In equation 45, $\tilde{g}(t)$ shows the estimation error. Also, ε_f equation 46 represents the error between the optimum estimate and the actual value of g.

There exists a positive value such that:

$$\left(\frac{\varepsilon_f sgn(\phi)}{a} - a\right)^2 \geq 0$$

$$\frac{\varepsilon_f^2 sgn^2(\phi)}{a^2} - 2\varepsilon_f sgn(\phi) + a^2 \geq 0$$

$$\varepsilon_f sgn(\phi) \leq \frac{1}{2}\left(\frac{\varepsilon_f^2 sgn^2(\phi)}{a^2} + a^2\right)$$

$$\varepsilon_f sgn(\phi) \leq \frac{\varepsilon_f^2 sgn^2(\phi)}{2a^2} + \frac{a^2}{2} \tag{47}$$

Considering equation 47 and assumptions 1, 3, and 4, we have:

$$\leq \phi(t)\left[b(x,t) - (\theta - \theta^*)^T \xi(X) sgn(\phi) - \varepsilon_f sgn(\phi) - \bar{b}\, sgn(\phi) - \eta\, sgn(\phi)\right] + \frac{1}{z}(\theta - \theta^*)^T\dot{\theta} \tag{48}$$

$$\leq \phi(t)\left[b(x,t)-(\theta-\theta^*)^T\xi(X)sgn(\phi)-\frac{\varepsilon_f^2 sgn(\phi)^2}{2a^2}-\frac{a^2}{2}-\bar{b}\,sgn(\phi)-\eta\,sgn(\phi)\right]+\frac{1}{z}(\theta-\theta^*)^T\dot{\theta} \tag{49}$$

$$\leq \phi(t)\left[b(x,t)-(\theta-\theta^*)^T\xi(X)sgn(\phi)-\frac{\varepsilon_f^2}{2a^2}-\frac{a^2}{2}-\bar{b}\,sgn(\phi)-\eta\,sgn(\phi)\right]+\frac{1}{z}(\theta-\theta^*)^T\dot{\theta} \tag{50}$$

$$= b(x,t)\phi(t)+\left[(\theta-\theta^*)^T\left(-\xi(X)|\phi|+\frac{1}{z}\dot{\theta}\right)\right]-\frac{\varepsilon_f^2}{2a^2}-\frac{a^2}{2}-\bar{b}|\phi|-\eta|\phi| \tag{51}$$

$$\dot{\theta}=z\xi(X)|\phi| \tag{52}$$

$$\leq -\eta|\phi|\leq 0 \tag{53}$$

$$\frac{1}{2}\frac{d}{dt}\phi(t)^2=\frac{1}{2}\frac{d}{dt}|\phi(t)^2|\leq -\eta|\phi(t)|,\quad \phi(0)\neq 0 \tag{54}$$

Based on equation 51, we can obtain the adaption rule as $\dot{\theta}=z\xi(X)|\phi|$.

6.2. Examining the Proposed Controller with Uncertainties in the SEIR Model

In this section, we devise a sliding mode controller for the SEIR model in the presence of uncertainties. The equations needed for this design are mentioned in (Ibeas, A., Shafi, M., Ishfaq, M., & Ali, M., 2017):

$$\dot{S}(t)=-\mu S(t)+\omega R(t)-\varphi(S,E,I,R)+\nu N(t)-N(t)V(t)+d_1 \tag{55}$$

$$\dot{E}(t)=\varphi(S,E,I,R)-(\mu+\sigma)E(t)+d_2 \tag{56}$$

$$\dot{I}(t)=-(\mu+\gamma)I(t)+\sigma E(t)+d_3 \tag{57}$$

$$\dot{R}(t)=-(\mu+\omega)R(t)+\gamma I(t)+N(t)V(t)+d_4 \tag{58}$$

Where d_1 to d_4 are systems' uncertainties and are defined as follows (note that Δ represents parameter uncertainty), other parameter definitions are the same as those shown in equations 1 to 4.

$$\begin{cases} d_1=-\Delta\mu\,S(t)+\Delta w\,R(t)+\Delta v\,N(t)\\ d_2=(-\Delta\mu-\Delta\sigma)E(t)\\ d_3=(-\Delta\mu-\Delta\gamma)I(t)+\Delta\sigma\,E(t)\\ d_4=(-\Delta\mu-\Delta w)R(t)+\Delta\gamma\,I(t)\end{cases} \tag{59}$$

The uncertainty of the system is limited such that

$$\begin{array}{ll} d_1 \in \Omega_1(\Delta\mu, \Delta v, \Delta w) & : \{d_1 min \le d_1 \le d_1 max\} \\ d_2 \in \Omega_2(\Delta\mu, \Delta\sigma) & : \{d_2 min \le d_2 \le d_2 max\} \\ d_3 \in \Omega_3(\Delta\mu, \Delta\sigma, \Delta\gamma) & : \{d_3 min \le d_3 \le d_3 max\} \\ d_4 \in \Omega_4(\Delta\mu, \Delta\gamma, \Delta w) & : \{d_4 min \le d_4 \le d_4 max\} \end{array} \tag{60}$$

d_1 to d_4 uncertainty terms are time functions (e.g., days, months, and years). In addition, these terms depend on μ: mortality rate resulting from natural causes, v: birth rate, and w: loss of health rate. Therefore, these three terms depending on μ ،v و w can not be greater than the population at all times. The whole population is known at all times. Thus, the upper limit of the population is assumed to be known at any given time. Consequently, we can assume that the upper limits of d_1 to d_4 uncertainties are known. Another assumption is that the lower limits of these uncertainties are near zero, which is trivial to proving the control system's stability. In assumption 5, $d_i min$ and $d_i max$ show the lower and upper limits of uncertainties, respectively. New dynamic changes become, adding equations 55 to 58 as follows:

$$\dot{N}(t) = (\nu - \mu)N(t) + d_1 + d_2 + d_3 + d_4 \tag{61}$$

Substituting equation 59 in 61, we can rewrite the dynamic population change as below:

$$\dot{N}(t) = (\nu - \mu)N(t) + (\Delta\nu - \Delta\mu)N(t) \tag{62}$$

Therefore, the sum of uncertainties can be expressed as $d_1 + d_2 + d_3 + d_4 = D = (\Delta\nu - \Delta\mu)N(t)$.

Now we prove the fuzzy sliding mode controller stability with uncertainties in the SEIR model.

$$\begin{aligned} \dot{\phi}(t) &= \dot{e}(t) + \lambda e(t) \\ &= \dot{R}(t) - \dot{N}_{ref}(t) + \lambda R(t) - \lambda N_{ref}(t) \\ &= -(\mu + \omega)R(t) + \gamma I(t) + N(t)V(t) + d4 - (\nu - \mu)N(t) - d_1 - d_2 - d_3 \\ &\quad - d4 + \varepsilon e^{-\varepsilon t}\big(R(0) - N(0)\big) + \lambda R(t) - \lambda N(t) - \lambda e^{-\varepsilon t}\big(R(0) - N(0)\big) \end{aligned} \tag{63}$$

For simplicity, we assume ε = λ, which removes parts with initial values.

$$\begin{aligned} &= -(\mu + \omega)R(t) + \gamma I(t) + N(t)V(t) - (\nu - \mu)N(t) - d_1 - d_2 - d_3 \\ &\quad + \lambda R(t) - \lambda N(t) = 0 \end{aligned} \tag{64}$$

From the above equation, the new equivalent control rule for the SEIR model with uncertainties is calculated as below:

$$\begin{aligned} V_{eq.new}(t) = \frac{1}{N(t)}[(\lambda + \nu - \mu)N(t) + (\mu + \omega - \lambda)R(t) - \gamma I(t) + (d_1 + d_2 \\ + d_3)] \end{aligned} \tag{65}$$

$$V_{eq.new} = V_{eq}(t) + \frac{1}{N(t)}(d_1 + d_2 + d_3) \tag{66}$$

Using equations 32 and 33, we can rewrite equation 66 as follows:

$$V_{eq.new}(t) = \frac{1}{N(t)}\left[\hat{\delta}N(t) + \hat{\rho}R(t) - \hat{\gamma}I(t) + (d_1 + d_2 + d_3)\right] \tag{67}$$

Assumption 6: we can define the parameter D_f such that $D_f \cong d_1 + d_2 + d_3$ and $d_1min + d_2min + d_3min \ \le D_f \le \ d_1max + d_2max + d_3max$

Assumption 7: considering assumption 3, we can chose g(x,t) as:

$$g_d(x,t) = g(x,t) - \left(\frac{D_f}{sgn\big(\phi(t)\big)}\right) \tag{68}$$

Such that η is a positive value chosen by the designer and $\eta > D_f$.

Putting equations 67 and 34 together, we write the general control rules as follows:

$$V_{new} = V_{eq}(t) - \frac{g_d(x,t)}{N(t)} sgn\big(\phi(t)\big) \tag{69}$$

Finally, such as equation 39, we can consider the fuzzy sliding mode control rule as follows:

$$V_{new} = V_{eq}(t) - \frac{\hat{g}_d(x,t)}{N(t)} sgn\big(\phi(t)\big) \tag{70}$$

From equation 69, we can rewrite equation 43 as:

$$= \phi(t)\left[(\hat{\rho} - \rho)R(t) + (\gamma - \hat{\gamma})I(t) + \big(\hat{\delta} - \delta\big)N(t) - \hat{g}_d(x,t)sgn(\phi)\right] + \frac{1}{z}(\theta - \theta^*)^T\dot{\theta} \tag{71}$$

$$\tilde{g}(t) = (\theta - \theta^*)^T\xi(X) + \varepsilon_f \tag{72}$$

$$\varepsilon_f = g - g(\theta^*|\xi(X)) \tag{73}$$

There exist a positive value a such that

$$\left(\frac{\varepsilon_f sgn(\phi)}{a} - a\right)^2 \ge 0$$

$$\frac{\varepsilon_f^2 sgn^2(\phi)}{a^2} - 2\varepsilon_f sgn(\phi) + a^2 \ge 0$$

$$\varepsilon_f sgn(\phi) \le \frac{1}{2}\left(\frac{\varepsilon_f^2 sgn^2(\phi)}{a^2} + a^2\right)$$

$$\varepsilon_f sgn(\phi) \le \frac{\varepsilon_f^2 sgn^2(\phi)}{2a^2} + \frac{a^2}{2} \tag{74}$$

Based on equation 74 and considering assumptions 1, 3, 4, and 7, we have:

$$\le \phi(t)\left[b(x,t) - (\theta - \theta^*)^T\xi(X)sgn(\phi) - \varepsilon_f sgn(\phi) - \bar{b}\ sgn(\phi) - \eta\ sgn(\phi) + D_f\ \right] + \frac{1}{z}(\theta - \theta^*)^T\dot{\theta} \tag{75}$$

$$\le \phi(t)\left[b(x,t) - (\theta - \theta^*)^T\xi(X)sgn(\phi) - \frac{\varepsilon_f^2 sgn(\phi)^2}{2a^2} - \frac{a^2}{2} + D_f\right.$$

$$- \bar{b}\, sgn(\phi) - \eta\, sgn(\phi)\Big] + \frac{1}{z}(\theta - \theta^*)^T \dot{\theta} \tag{76}$$

$$\leq \phi(t)\left[b(x,t) - (\theta - \theta^*)^T \xi(X) sgn(\phi) - \frac{\varepsilon_f^2}{2a^2} - \frac{a^2}{2} - \bar{b}\, sgn(\phi) - \eta\, sgn(\phi)\right] + \frac{1}{z}(\theta - \theta^*)^T \dot{\theta} \tag{77}$$

$$= b(x,t)\phi(t) + \left[(\theta - \theta^*)^T\left(-\xi(X)|\phi| + \frac{1}{z}\dot{\theta}\right)\right] - \frac{\varepsilon_f^2}{2a^2} - \frac{a^2}{2} + D_f - \bar{b}|\phi| - \eta|\phi| \tag{78}$$

$$\dot{\theta} = z\xi(X)|\phi| \tag{79}$$

$$\leq -\eta|\phi| \leq 0 \tag{80}$$

$$\frac{1}{2}\frac{d}{dt}\phi(t)^2 = \frac{1}{2}\frac{d}{dt}|\phi(t)|^2 \leq -\eta|\phi(t)|, \quad \phi(0) \neq 0 \tag{81}$$

As can be seen from the above equations, the derivative of the Lyapunov function is negative. Thus, based on Lyapunov theory, the system has reached stability. Also, the adaption rule is obtained as $\dot{\theta} = z\xi(X)|\phi|$, based on equation 78.

7. Simulation

Some simulation results are provided in this section to clarify the capabilities of the proposed controller. After that, a comparison is made between the results of the proposed controller and those proposed in [2]. This comparison entails two parts; one with uncertainties in the SEIR model and one without uncertainties in the SEIR model.

7.1. SEIR Epidemic Model Fuzzy Sliding Mode Controller without Uncertainties

This subsection compares the results from fuzzy sliding mode controller simulations to those of classic sliding mode controllers. No uncertainty is included. The simulation parameters are derived from [2].

The actual parameters of the system are as follows:

$$\mu^{-1} = 255\ days \qquad \sigma^{-1} = 1.2\ days \qquad \omega^{-1} = 12\ days$$
$$\nu^{-1} = 115\ days \qquad \beta = 1.66\ days^{-1} \qquad \gamma = \sigma \tag{82}$$

Nominal parameters are considered as follows:

$$\hat{\mu}^{-1} = 255\ days \qquad \hat{\sigma}^{-1} = 1.2\ days \qquad \hat{\omega}^{-1} = 12\ days$$
$$\hat{\nu}^{-1} = 115\ days \qquad \hat{\beta} = 1.66\ days^{-1} \qquad \hat{\gamma} = \hat{\sigma} \tag{83}$$

Also, controlling parameters are as below:

$$\lambda = \varepsilon = \frac{1}{2} days^{-1} \tag{84}$$

The fuzzy system uses individual fuzzified, multiplication inference, and centers' mean defuzzifire. The membership functions are defined to be Gaussian.

Figures 4 and 5 show the closed loop output of the proposed controller and the classic sliding mode one, respectively. In these figures, as stated earlier, the R-curve expresses the subpopulation recovered from or is immune to the epidemic disease. Accordingly, S denotes the susceptible subpopulation, E is the exposed subpopulation, and I the infectious subpopulation. As seen, susceptible, exposed, and infectious populations vanish entirely in less than 10 days. The recovered and immune population, however, increases with population dynamic change. As it is apparent from these figures, the two controllers can eliminate the epidemic disease. However, the proposed controller performance is improved to some extent.

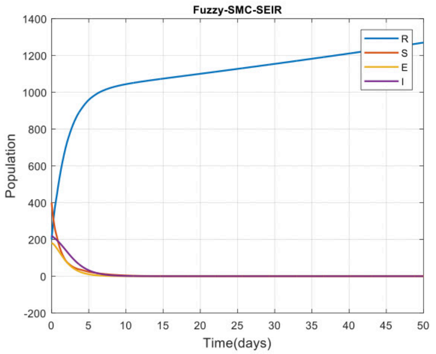

Fig. 4. closed loop output for fuzzy sliding mode controller

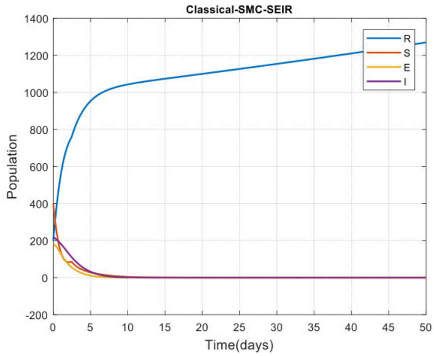

Fig. 5. closed loop output for classic SMC

The vaccination control signal is shown in figures 6 and 7. As seen in figure 6, the vaccination control signal for the fuzzy sliding mode controller is smoother. It has fewer fluctuations than the classic sliding mode controller, which means that the proposed controller is more optimal than the classic one in the vaccination plan.

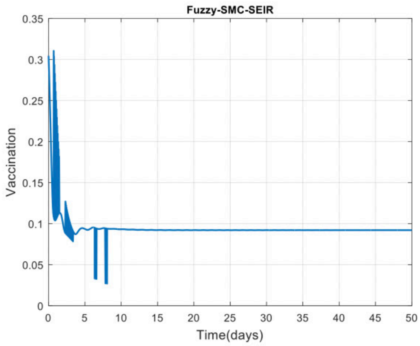

Fig. 6. Vaccination control signal for fuzzy SMC

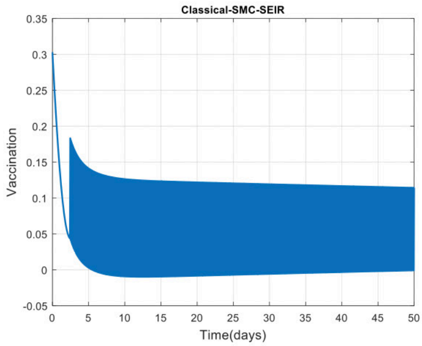

Fig. 7. Vaccination control signal for classic SMC

Tracking error for both proposed and classic sliding mode is shown in figures 8 and 9, respectively. As was mentioned earlier, N shows the total population, and N_{ref} expresses the desired trend for the immune population, or in other words, the reference signal. As stated in (8), tracking error ($e(t)$) is the difference between recovered people, $R(t)$, and N_{ref}.

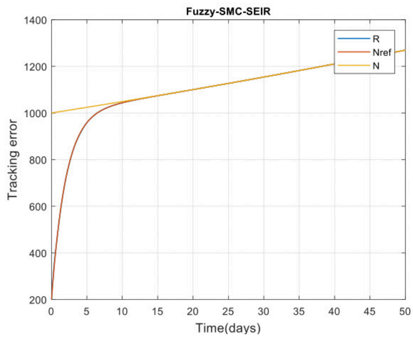

Fig. 8. Tracking error for fuzzy SMC

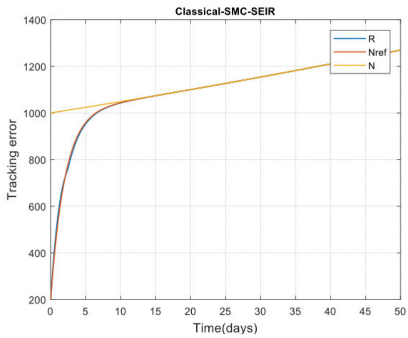

Fig. 9. Tracking error for classic SMC

Figures 10 to 13 show tracking error magnification in different time intervals. It can be seen from figure 10 that while the disease is not eradicated, the proposed controller includes fewer tracking errors. Moreover, observed from figures 12 and 13, we can conclude that when the disease is eradicated, the proposed controller performs better tracking with fewer ripples compared to the classic sliding mode controller. As can be seen, there is no chattering because of applying fuzzy logic to the sliding mode control law.

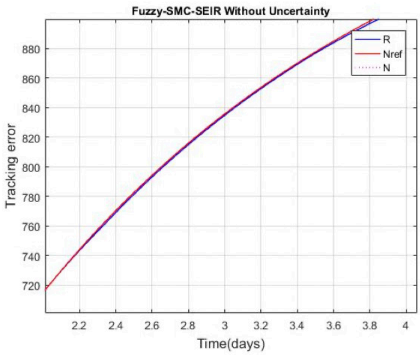

Fig. 10. Fuzzy SMC tracking error magnification (before eradication)

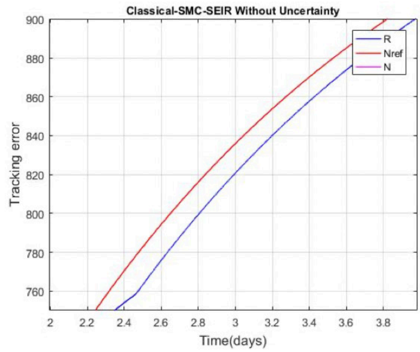

Fig. 11. Classic SMC tracking error magnification (before eradication)

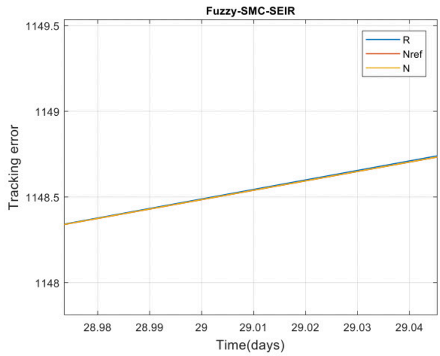

Fig. 12. Fuzzy SMC tracking error magnification (after eradication)

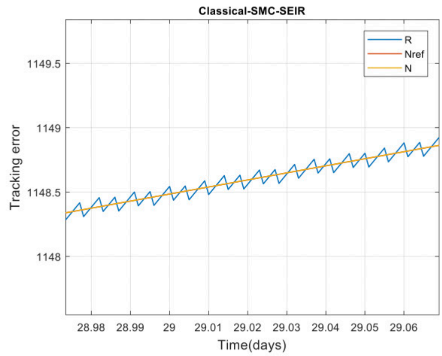

Fig. 13. Classic SMC tracking error magnification (after eradication)

7.2. SEIR Epidemic Model Fuzzy Sliding Mode Controller with Uncertainties

In this subsection, the simulation results of both fuzzy sliding mode and classic sliding mode controllers are provided for the SEIR model in the presence of the uncertainty sentence, expressed in (Ibeas, A., Shafi, M., Ishfaq, M., & Ali, M., 2017), and the results are compared.

The model parameters are similar to the ones from the previous subsection. Other parameters relating to uncertainty are taken from (Ibeas, A., Shafi, M., Ishfaq, M., & Ali, M., 2017) as in the following:

$$\Delta\mu^{-1} = 20 \text{ days} \qquad \Delta w^{-1} = 2.5 \text{ days} \qquad \Delta v^{-1} = 15\, days \qquad \Delta\sigma^{-1} = \Delta\gamma^{-1} = 1\, days \tag{85}$$

Figures 14 and 15 depict the closed-loop output of the system for classic and proposed sliding mode controllers with the presence of uncertainty in the SEIR model, respectively. It can be seen that susceptible, exposed, and infectious populations vanish in less than 10 days. The recuperated and immune population increases concerning changes in population dynamics.

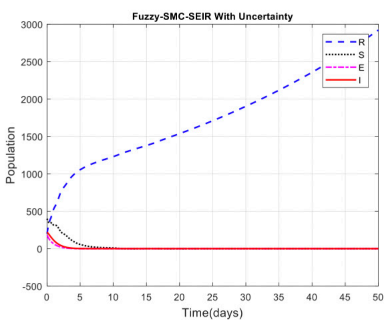

Fig. 14. Closed-loop output for fuzzy SMC in the presence of uncertainty in the model

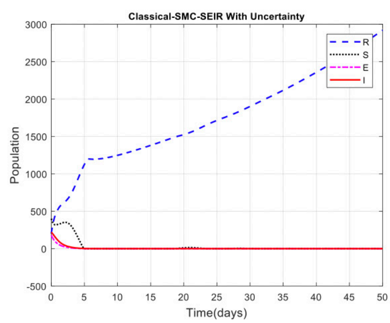

Fig. 15. Closed-loop output for classic SMC with the presence of uncertainty in the model

It is also apparent that the added uncertainty affects the performance of the classic sliding mode controller, resulting in worsened output compared to other states. Also, the proposed controller is more robust in the presence of uncertainty compared to the classic controller.

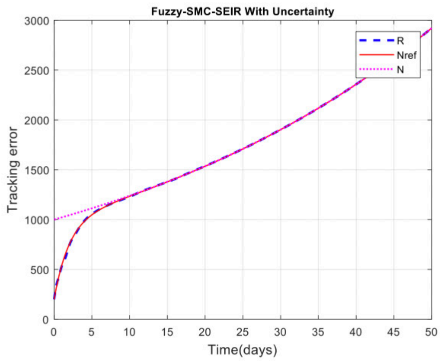

Fig. 16. Tracking error signal for the fuzzy SMC in the presence of uncertainty in the model

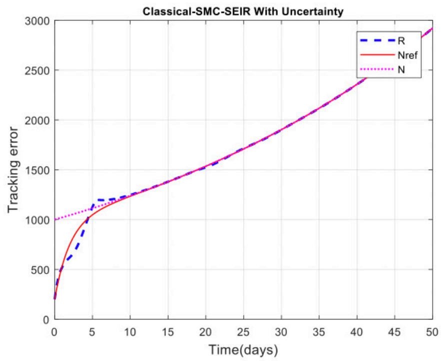

Fig. 17. Tracking error signal for the classic SMC in the presence of uncertainty in the model

Recuperated individuals' signals and the reference signal for the classic sliding mode controller are graphed in figure 16. This figure also shows the tracking error in different time intervals. Also, compared to the classic sliding mode controller without uncertainty, the tracking error at the beginning is more significant. Figure 17 illustrates the tracking error for fuzzy SMC in the presence of uncertainty. Proper tracking of the reference signal emphasizes the efficiency and robustness of the controller.

The vaccination control signal for the classic SMC in the presence of uncertainty is shown in figure 18. Based on the graph shown in figure 19, the fuzzy SMC has fewer ripples and chattering compared to the classic SMC and reaches stability

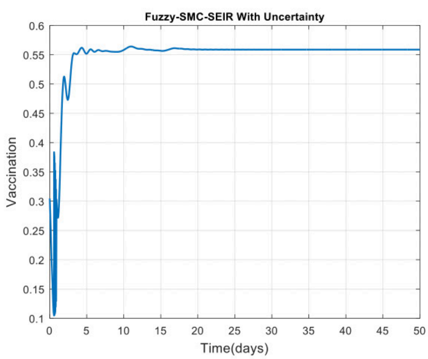

Fig. 18. Vaccination control signal for the fuzzy SMC in the presence of uncertainty

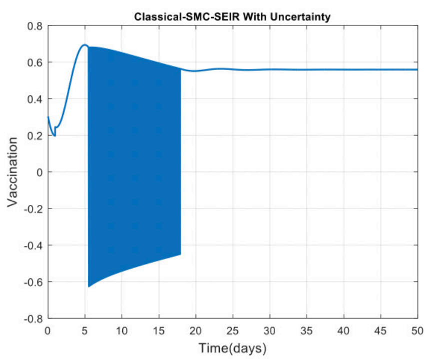

Fig. 19. Vaccination control signal for the classic SMC in the presence of uncertainty

The tracking error can be considered as a quantity to evaluate controller performance. Table 1 shows the difference between the maximum absolute error and the mean squared error in the classic sliding mode controller and fuzzy sliding mode controller, both in the presence and absence of the uncertainty sentence in the SEIR epidemic model.

Table 1: Error difference between fuzzy SMC and classic SMC

	Maximum absolute error in the absence of uncertainty sentence	Maximum absolute error in the presence of uncertainty sentence	mean squared error in the absence of uncertainty sentence	mean squared error in the presence of uncertainty sentence
The classic sliding mode controller	32.05	162.38	30.93	1359.8
The fuzzy sliding mode controller	11.47	36.48	1.05	24.03

8. Conclusion

This paper proposed a fuzzy sliding mode controller for epidemic diseases based on the SEIR model. Various types of vaccination rules based on control theory have been proposed and can be found in Classic sliding mode control can achieve the control objective and is capable of eradicating epidemic disease. However, all these works consider some parameters to determine infectious diseases. These parameters include birth, mortality, and prevalence rates. This situation is relatively unrealistic since estimating such parameters based on nominal and empirical data might result in incorrect values. Therefore, the vaccination rule is miscalculated, and its usage in the natural system might preclude the desired eradication of the disease. Moreover, the chattering phenomenon in the control signal of the classic sliding mode controller makes the implementation of the existing system impossible.

Utilizing a fuzzy sliding mode controller and online parameter tuning by adaption rule reduces the tracking error. Compared to the classic sliding mode controller, it eliminates the chattering phenomenon in the control signal. This makes it possible to reach the control objective without having to account for parametric uncertainty. Afterward, and by exerting the parametric uncertainty terms to the SEIR epidemic model, it was shown that the proposed fuzzy SMC has better performance, less tracking error, and more robustness than the classic SMC. Also, the Lyapunov theory was used to ensure the system's stability. As an affirmation, the simulation results illustrated the eradication of the epidemic disease by showing susceptible, exposed, and diseased populations' convergence towards zero. The type-2 fuzzy controller can be used in the future to obtain a better approximation. The proposed controller can also be extended to be applied to COVID-19 or delayed SEIR models.

References

Alonso-Quesada, S., De la Sen, M., Agarwal, R. P., & Ibeas, A. (2012). An observer-based vaccination control law for an SEIR epid*emic model based on feedback linearization techniques for nonlinear systems. Advances in Difference Equations, 2012(1), 1-32.*

*Bai, Z., & Zhou, Y. (2*012). Global dynamics of an SEIRS epidemic model with periodic vaccination and seasonal contact rate. Nonlinear Analysis: Rea*l World Applications, 13(3), 1060-1068.*

*Bart*oszewicz, A. (1995). A comment on 'A time-varying sliding surface for fast and robust tracking control of second-order uncertai*n systems'. Automatica, 31(12), 1893-1895.*

*Ch*eng, Y., Pan, Q., & He, M. (2013). Disease control of delay SEIR model with nonlinear incidence rate and vertical transmiss*ion. Computational and Mathematical Methods in Medicine,* 2013.

Di Giamberardino, P., & Iacoviello, D. (2017). Optimal control of SIR epidemic model with state dependent switching cost index. *Biomedical signal processing and control, 31,* 377-380.

Dong, N. P., Long, H. V., & Khastan, A. (2020). Optimal control of a fractional order model for granular SEIR epidemic with uncertainty. *Communications in nonlinear science and numerical simulation, 88, 105312.*

Edwards, C., & Spurgeon, S. (1998). Sliding mode control: theory and applications. Crc Press.

G. Drakopoulos, P. Mylonas and S. Sioutas (2019). A Case of *Adaptive Nonlinear System Identification w*ith Third Order Tensors in TensorFlow. In *IEEE International Symposium on INnovations in Intelligent SysTems and Applications (INISTA),* 1-6.

Hung, *J. Y., Gao, W., & Hung, J. C. (1993). Variable structure c*ontrol: A survey. *IEEE transactions on industrial electronics, 40*(1), 2-22.

Ibeas, A., De La Sen, M., & Alonso-Quesada, S. (2014). Robust sliding control of SEIR epidemic models. *Mathematical Problems in Engineering,* 2014.

Ibeas, A., de la Sen, M., & Alonso-Quesada, S. (2014, December). Adaptive control of SEIR discrete-time epidemic models. In AIP Conference Proceedings (Vol. 1637, No. 1, *pp. 37-46). American Institute of Physi*cs.

Ibeas, A., de la Sen, M., Alonso-Quesada, S., & Nistal, R. (2015, May). Partial stability-based vaccination control of SEIR epidemic models. In 2015 10th Asian Control Conference (ASCC) (pp. 1-6). IEEE.

Ibeas, A., Shafi, M., Ishfaq, M., & Ali, M. (2017). Vaccination controllers for SEIR epidemic models based on fractional order dynamics. Biomedical Signal Processing and Control, 38, 136-142.

Jang, J. S. (1992, March). Fuzzy controller design without domain experts. In [1992 Proceedings] IEEE International Conference on Fuzzy Systems (pp. 289-296). *IEEE.*

Jiao, H., & Shen, Q. (2020). Dynamics analysis and vaccination-based sliding mode control of a more generalized SEIR epidemic model. *IEEE Access, 8,* 174507-174515.

Lee, H., Kim, E., Kang, H. J., & Park, M. (1998). Design of a sliding mode controller with fuzzy sliding surfaces. *IEE Proceedings-Control Theory and Applications, 145*(5), 411-418.

Lee, H. J. (2022). Robust static output-feedback vaccination policy design for an uncertain SIR epidemic model with disturbances: *Positive Takagi–Sugeno model approach. Biomedical Signal Processing and Control, 72,* 103273.

Manthouri, M., Aghajari, Z., & Safary, S. (2022). Computational Intelligence Method for Detection of White Blood Cells Using Hybrid of Convolutional Deep Learning and SIFT. *Computational and Mathematical Methods in Medicine, 2022.*

Mattsson, P., Zachariah, D., & Stoica, P. (2018). *Recursive nonlinear-system identification using latent variables. Automatica, 93*, 343-351.

McCluskey, C. C. (2010). Complete global stability for an SIR epidemic model with delay—distributed or discrete. Nonlinear Analysis: *Real World Applications, 11*(1), 55-59.

Ohtake, H., Tanaka, K., & Wang, H. O. (2006). Switching fuzzy controller design based on switching Lyapunov function for a class of nonlinear systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 36*(1), 13-23.

Schoukens, J., & Ljung, L. (2019). Nonlinear system identification: A user-oriented road map. *IEEE Control Systems Magazine, 39*(6), 28-99.

Tokat, S., Eksin, I., & Güzelkaya, M. (2003). New approaches for on-line tuning of the linear sliding surface slope in sliding mode controllers. *Turkish Journal of Electrical Engineering and Computer Sciences, 11(1),* 45-60.

Wu, S. J., & Lin, C. T. (2000). Optimal fuzzy controller design: local concept approach. *IEEE Transactions on Fuzzy Systems, 8(2), 1*71-185.

Yang, F., Liu, H., Qi, H., & Liu, X. (2016, December). SEIR evolutionary simulation model of the infectious disease emergency. In *2016 International Conference on Industrial Informatics-Computing Technology, Intelligent Technology, Industrial Information Integration (ICIICII) (pp. 315-318). IEEE.*

Yi, N., Zhang, Q., Mao, K., Yang, D., & Li, Q. (2009). Analysis and control of an SEIR epidemic system with nonlinear transmission rate. *Mathematical and computer modelling, 50(9*-10), 1498-1513.

Submitted: 28.01.2022
Accepted: 24.05.2022